TRANSNATIONAL COORDINATION FAILURES IN INTERTEMPORAL COUNTERTERRORISM GAMES

João Ricardo Faria and Emilson C. D. Silva

WP2011-006
November 17, 2011

http://www.econ.gatech.edu/research/workingpapers

School of Economics
Georgia Institute of Technology
221 Bobby Dodd Way
Atlanta, GA 30332-0615

Transnational Coordination Failures in Intertemporal Counterterrorism Games

João Ricardo Faria and Emilson C. D. Silva

WP2011-006

November 17, 2011

JEL: C79, D74, F59, H87, H23

ABSTRACT

This paper fills an important gap in the literature. It is the first systematic effort of addressing counterterrorism policy coordination failures due to transnational intertemporal externalities. As these externalities involve both spatial and time dimensions, non-cooperative policy coordination failures are better captured in a framework that allows us to consider two types of non-cooperative dynamic games, one in which national authorities are myopic and another in which they are farsighted. We show that the steady state outcomes for both types of non-cooperative games are characterized by larger counterterrorism expenditures than their counterparts in the social optimum. The farsighted equilibrium always yields greater levels of counterterrorism expenditures, terrorist activities and violence than those produced by the myopic equilibrium. Thus, the distortion produced by the farsighted equilibrium is greater than the distortion produced by the myopic equilibrium.

João Ricardo Faria

MPA Program, University of Texas at El Paso

500 West University Avenue, El Paso, Texas, 79968-0703

Email: rfaria2@utep.edu

Emilson C. D. Silva

School of Economics, Georgia Institute of Technology

221 Bobby Dodd Way, Atlanta, GA 30332-0615

E-mail: emilson.silva@econ.gatech.edu

Transnational Coordination Failures in Intertemporal Counterterrorism Games

The preeminent objective of bin Laden, explicit in some of his fatwas and messages (e.g. bin Laden et al. (1998)), is the restoration of a pan-islamic caliphate (Hoffman (2006, p.96); see also Cook (2006) and Hashim (2006)). The caliphate is considered as the golden age of Islam, identified historically as Islam's apex as a socio-economic and cultural system. The utilization of historical references as motivators for current and future actions clearly situates al-Qaeda as a terrorist organization which displays history-dependent preferences. This is also a common feature of other terrorist organizations. Psychologists call this phenomenon 'intertemporal organizational identification': an identification that promotes long-term legacy building. According to Fox et al (2010), the phenomenon of identification with past generations serve to facilitate the identification with future generations, since the connection between past and present are more easily clarified “than the connection between the present and the future" (Fox et al., 2010, p.175). Yet, psychological motivations to carry out terrorist activities cannot be easily separated from strategic considerations or motivations because psychological and strategic motivations are more likely to complement each other (see, e.g., Horgan (2005)). One of our main contributions is to show that strategic choices made by a terrorist organization featuring history-dependent preferences produce intertemporal terror externalities.

This paper is the first systematic effort of studying counterterrorism policy coordination failures due to transnational intertemporal externalities.[1] We analyze dynamic games between two long-lived national governments which fight a common, long-lived, strategic, terrorist

[1] The study of intertemporal externalities is well-developed in areas such as environmental and resource economics. Hartwick (1980), one of the pioneering papers in this literature, shows that market failure occurs with free access because an agent foregoing some “catch” today reaps in the future the discounted average (defined over the number of agents) rather than the marginal payoff.

organization. The key dynamic components of our model are fruits of the history-dependent preferences that characterize the behavior of the international terrorist organization. At any period, the terrorist organization derives complementary utility from past and contemporaneous levels of successful terrorist activities.[2] This implies that optimal (strategic) choices made by the terrorist organization link past, present and future terrorist activities, and the links can be properly captured by terrorist evolution equations. The national governments design counterterrorism policies subject to the terrorist evolution equations.

We study coordination failures associated with both spatial and time dimensions of the national authorities' non-cooperative decision making. We consider three different games: one game in which the nations fully coordinate their counterterrorism policies and two games in which there is no coordination. In the first uncoordinated game, each nation is myopic in the sense that its decision making accounts for the evolution of terrorist activity that occurs within national boundaries only. In the second uncoordinated game, each nation is farsighted in the sense that its decision making accounts for the evolution of terrorist activities in both nations and it internalizes the feedback effects that its counterterrorism effort promotes one the other nation's counterterrorism effort. Since a nation's decision regarding counterterrorism action today affects the other nation today as well as in the future, the farsighted nations, unlike the myopic nations, formulate counterterrorism policies by accounting for both current and future joint feedback effects of their counterterrorism actions. A myopic nation neglects both current and future feedback effects produced by the other nation's reactions to its policy making.

The paper is organized as follows. Section 2 reviews the literature. Section 3 presents a simple model for one nation, which focuses on the dynamics emerging from the strategic choices

[2] In this sense, our model is consistent with economic models of habit formation (see, e.g., Ryder and Heal (1973), Boyer (1978), Boyer (1983), Iannaccone (1986), Becker and Murphy (1988), Dockner and Feichtinger (1993)). See also Rozen (2010).

made by a terrorist organization characterized by history-dependent preferences and the optimal defensive counterterrorism policy implemented by a national government. After the key properties of the simple model are fully understood, in section 4 we extend it to two nations. In this extension, we capture the spatial (transnational) dimension of the externalities. We first examine the coordinated policy game, which is our efficient benchmark allocation. We later compare the results for the coordinated equilibrium with the results for the uncoordinated myopic equilibrium and the results for the uncoordinated farsighted equilibrium. We also compare the results for the two types of uncoordinated equilibrium. Discussion and concluding remarks are offered in section 5.

2. Literature Review

This paper investigates the key theoretical properties of transnational intertermporal externalities in dynamic counterterrorism games. A transnational externality exists if one nation's action imposes uncompensated costs or benefits on one or more nation(s). If one nation's action in the present imposes a cost or benefit on one or more nation(s) in the future, and no compensation is received or paid, then there is a transnational intertemporal externality (Enders and Sandler,2006, pp.140-141).

It is well-known that counterterrorism policies may give rise to transnational external benefits and costs. For instance, defensive counterterrorism actions, such as installation of metal detectors in airports, can create external costs by deflecting terrorist attacks from the country with metal detectors in airports to other countries with unprotected airports. Proactive counterterrorism, on the other hand, can generate external benefits. For example, a country that attacks and reduces the resources of a terrorist organization that is a common threat to other countries generates an external benefit to them (Sandler, 2005).[3]

When a nation grants concessions to hostage takers it creates an intertemporal external cost to other countries, since terrorists believe that future hostage taking will be successful and increasingly profitable (Lapan and Sandler, 1993). If, however, instead of negotiating and granting concessions, the country applies a successful procedure for freeing hostages then it generates an intertemporal external benefit.

Externalities explain the collective action failures that plague countries fighting global terrorism. Since some proactive policies may provoke a terrorist backlash (Rosendorff and Sandler 2004; Siqueira and Sandler 2007; Faria and Arce 2011a) countries are less likely to cooperate among themselves and undersupply proactive counterterrorism; they rather resort to unilateral defensive counterterrorism, oversupplying it (Lee 1988; Lee and Sandler 1989; Sandler and Lapan 1988; Arce and Sandler 2005; Sandler and Siqueira 2006). These are standard results in static games.[4]

Our setup is an infinite horizon dynamic game played by two long-lived national governments which fight a long-lived international terrorist organization. Dynamic games in terrorism have been studied, among others, by Faria (2003), Feichtinger and Novak (2008) and Novak et al. (2010). Faria (2003) provides a pioneering theoretical framework to explain terror cycles.[5] He examines a Stackelberg differential game in which terrorists are followers and the government is the leader. The dynamic optimization problem of terrorists yields a time path for terrorist attacks that is considered as an additional dynamic constraint for the government. The

[3] Feldstein (2008, p. 126) notes that countries have an incentive to collaborate with the US because of its superior space-based technology for surveillance of electronic communication by voice and internet and for photographic reconnaissance.

[4] See Sandler and Arce (2003, 2007) for surveys on game theoretical approaches to terrorism.

[5] See also Das (2008) for a model of terrorist cycles. Behrens et al. (2007) examine a Stackelberg dynamic game of international terrorism.

government maximizes national security subject to the evolution of law enforcement and the optimal reaction of terrorists. The model yields a limit cycle between government counterterrorism efforts and terrorist activities.

In Feichtinger and Novak (2008), the differential game between government and a terrorist organization is simultaneous. Both are subjected to the non-linear growth function dynamics of terrorist resources. In the open-loop Nash solution, they show that if terrorists are more impatient than the government, the system (terrorist resources, government counterterrorism, and terrorist attacks) may exhibit long-run persistent oscillations. In a closely related framework, Novak et al. (2010) consider a state-separable differential game. Given specific parameter values, they show that in the Stackelberg equilibrium terrorists act more cautiously and the government more aggressively in comparison to their behaviors in the Nash equilibrium.

Our framework differs from the papers that examine dynamic terror games in (at least) three significant ways. First, the dynamics emerge from history-dependent preferences exhibited by the international terrorist organization. Second, we consider spatial and intertemporal externalities arising with uncoordinated counterterrorism policy making. Third, we consider myopic and farsighted counterterrorism policy making.

3. Intertemporal Terror Game in a Single Nation

We consider an economy with an infinite time horizon, $t = 0,1,...,\infty$. There are two types of long-lived players, the government and a terrorist organization.

The government has the mission of minimizing the present value of the social loss function associated with terrorist attacks. We assume that the government is perfectly informed about the terrorist organization's technology and payoff functions and that, in each period, it can perfectly observe and nullify a fraction of the quantity of terrorist attacks (i.e., the terrorist organization's output) before they cause violence. The government can also perfectly observe the remaining fraction of the quantity of terrorist attacks that succeed in producing violence in each period. However, the government cannot observe the terrorist organization's covert productive and managerial activities (e.g., recruitment, logistic and strategic planning, etc.). Thus, the government faces moral hazard vis-à-vis the myriad of organizational actions that the terrorist organization carries out to produce attacks.[6]

Let $a_t \geq 0$ denote the quantity of attacks in period t. At any period t, a fraction $\sigma_t \in [0,1]$ of attacks are successful in causing violence. The quantity of successful attacks in period t is denoted $s_t = \sigma_t a_t$. Also, at any period t, the government exerts non-negative efforts in defensive counterterrorism activities.[7] Let $g_t \geq 0$ denote the level of effective defensive counterterrorism effort in period t. Defensive effort reduces the quantity of attacks in period t at a rate $\varphi_t \geq 0$; hence, $g_t = \varphi_t a_t$. If $a_t > 0$, we have $\sigma_t + \varphi_t = 1$.

We assume that the level of violence experienced by society in period t, v_t, is equal to the quantity of successful attacks; namely, $v_t = s_t$. If $a_t > 0$, $v_t = s_t = a_t - g_t$. The social harm associated with v_t units of violence in period t is $h(v_t)$, where we assume that $h(0) = h'(0) = h''(0) = 0$ and $h(v_t) > 0$, $h'(v_t) > 0$ and $h''(v_t) > 0$ for all $v_t > 0$. The monetary

[6] This is similar to the moral hazard problem faced by police enforcement which can detect and nullify some criminal activities but cannot observe the actions carried out by the criminals to "manufacture" the criminal activities. Also, in many instances, the identities of the criminals are unknown ex-ante and remain unknown ex-post.

[7] For tractability and in order to emphasize the intertemporal aspects of our model, we do not consider proactive counterterrorism efforts in this paper. The spillovers considered in the expanded model below emerge from defensive counterterrorism efforts. We are currently working on an extension of this paper's framework which incorporates proactive counterterrorism efforts.

cost of producing g_t units of counterterrorism effort is $c(g_t)$, where we assume that $c(0) = c'(0) = c''(0) = 0$ and $c(g_t) > 0$, $c'(g_t) > 0$ and $c''(g_t) > 0$ for all $g_t > 0$. The social loss in period t is the sum of the cost of providing counterterrorism effort and the damage caused by violence: $z(g_t, v_t) = c(g_t) + h(v_t)$. The present value of the social loss function is, therefore, $Z = \sum_{t=0}^{\infty} \rho^t z(g_t, v_t) = \sum_{t=0}^{\infty} \rho^t \left[c(g_t) + h(v_t) \right]$, where $\rho \equiv 1/(1+r)$ is the discount rate and $r > 0$ is the interest rate. For concreteness, and in order to facilitate comparisons, we shall assume henceforth that $c(g_t) = g_t^2$ and $h(v_t) = v_t^2$. These assumptions imply that the present value of the social loss function is $Z = \sum_{t=0}^{\infty} \rho^t \left(g_t^2 + v_t^2 \right)$.

We postulate that past successful attacks is one of the motivators for the planning and execution of attacks in any given period. Habit formation characterizes the behavior of the terrorist organization: at any period, it derives complementary utility from past and contemporaneous levels of successful attacks. As we shall see below, for a long-lived terrorist organization, habit formation implies that past, present and future acts of violence are interlinked.

Formally, in any period t, we assume that the terrorist organization's utility function is $u_t = u(s_t, s_{t-1}, y_t; g_t) = y_t + bs_t + (s_t + g_t) s_{t-1}$, $b > 0$, where $y_t \geq 0$ denotes the contemporaneous level of a composite good (i.e., numeraire) consumed by the terrorist organization. If $s_{t-1} > 0$, this formulation is consistent with economic models of habit formation and addiction (see, e.g., Becker and Murphy (1988), Boyer (1978), Boyer (1983), Dockner and Feichtinger (1993), Iannaccone (1986), Ryder and Heal (1973)) in that the intertemporal behavior of the terrorist organization features "adjacent complementarity." This complementarity follows from the facts that the marginal utility associated with the level of successful attacks in period t, $\partial u_t / \partial s_t = b + s_{t-1}$, is increasing in the level of successful attacks in period $t-1$ and the marginal utility associated with successful attacks in period $t-1$, when evaluated in terms of the utility at period t, $\partial u_t / \partial s_{t-1} = s_t + g_t$, is increasing in the level of successful attacks in period t. It is also important to note that the latter is increasing in the level of counterterrorism effort supplied by the government in period t. By including the level of counterterrorism effort in the terrorist organization's utility function, we are allowing for another potential type of intertemporal incentive to produce terror activity: if contemporaneous terror activity leads to an increase in the government's counterterrorism effort level in the next period, it will also lead to an increase in the social loss faced by the government due to the increase in the cost of combating terrorism. This is in accordance with the view that one of the key objectives of terrorist organizations is to weaken the government's economic position (see, e.g., bin Laden et al (1998) and Gartenstein-Ross (2011)). We, therefore, assume that the terrorist organization derives utility from violence and the economic cost faced by the government in combating terrorism, namely, from both components of the government's social loss function.

Note that the terrorist organization may find it advantageous to supply attacks in period t even if the government was successful in avoiding all violence attempts in period $t-1$ (i.e., $s_{t-1} = 0$).[8] However, it is not rational for the terrorist organization to supply attacks at period t if it expects that $s_t = s_{t-1} = 0$.

We assume that the technology employed by the terrorist organization is characterized by decreasing returns. This seems to be a realistic assumption due to the nature of the "terrorist

[8] In the games examined below, the terrorist organization makes its decisions after it observes governmental policy commitments. It is, therefore, able to calculate if it is optimal to supply attacks in any given period.

business," which involves a short-time perspective (since its operation must be constantly switching locations or tactics in order to prevent detection and apprehension) and there are various types of fixed or setup costs in recruiting, planning, attack preparation and attack execution.

Assuming that the terrorist organization finds it desirable to supply a positive level of attacks in period t (an assumption that holds in equilibrium, as we demonstrate below), the consumption level of the numeraire good in this period is $y_t = \bar{y} - \bar{f} - a_t^2/2$. We assume that $\bar{y} > \bar{f} \geq 0$, where $\bar{y}$ denotes the periodic amount of income available to the terrorist organization, $\bar{f}$ represents the periodic fixed cost the terrorist organization incurs to carry out its activities and $a_t^2/2$ is the variable operational cost. For notational simplicity, we assume that $\bar{f} = 0$ henceforth. We also assume that the terrorist organization is always capable of raising sufficient income to finance its variable costs and that this fact is common knowledge (i.e., the government knows this as well); namely, $\bar{y}$ is sufficiently large so that $\bar{y} \geq a_t^2/2$.

The present value of utility for the terrorist organization, if it supplies a positive level of attacks in each period, is

$$\sum_{t=0}^{\infty} \rho^t u\left(s_t, s_{t-1}, \bar{y} - a_t^2/2; g_t\right) = \bar{y}/(1-\rho) + \sum_{t=0}^{\infty} \rho^t \left[bs_t + s_{t-1}\left(s_t + g_t\right) - \left(s_t + g_t\right)^2/2 \right],$$

where $\bar{y}/(1-\rho) = \sum_{t=0}^{\infty} \rho^t \bar{y}$ and $\sum_{t=0}^{\infty} \rho^t\, a_t^2/2 = \sum_{t=0}^{\infty} \rho^t \left(s_t + g_t\right)^2/2$. This intertemporal utility function makes it clear that past, present and future benefits and costs associated with the supply of attacks are interlinked. These intertemporal linkages produce a legacy of terror.

In the infinite-horizon game played by the government and the terrorist organization, we assume that the government makes its intertemporal policy choices in full anticipation of the intertemporal choices made by the terrorist organization and that there is common knowledge. We can thus consider a setting in which the government announces its policy commitments, $\{g_t\}_{t=0,\ldots,\infty}$, and the terrorist organization makes its intertemporal choices, $\{a_t\}_{t=0,\ldots,\infty}$, after it observes the government's announcement.[9] We derive a subgame-perfect Nash equilibrium for the infinite-horizon game.

Having observed the government's announcement, $\{g_t\}_{t=0,\ldots,\infty}$, the terrorist organization makes its intertemporal choices to maximize $\sum_{t=0}^{\infty}\rho^t\left[bs_t+s_{t-1}(s_t+g_t)-(s_t+g_t)^2/2\right]$ subject to $a_{-1}=g_{-1}=0$, $s_t=a_t-g_t$ if $a_t>g_t$, $s_t=0$ if $a_t\le g_t$, $s_{t-1}=a_{t-1}-g_{t-1}$ if $a_{t-1}>g_{t-1}$ and $s_{t-1}=0$ if $a_{t-1}\le g_{t-1}$. As we shall demonstrate below the first order conditions for the optimization problems solved by both players ensure that $a_t>g_t>0$ for all t in the steady state. Assuming that $s_{t-1}>0$ and $s_t>0$, the first order conditions for the current problem with respect to a_t and a_{t+1} yields

$$a_t-\rho a_{t+1}=b+a_{t-1}-g_{t-1} \tag{1a}$$

$$-a_t+a_{t+1}=b+\rho a_{t+2}-g_t\,. \tag{1b}$$

The linear system of equations (1a) and (1b) can be written in matrix form as follows:

$$\begin{pmatrix}1 & -\rho\\ -1 & 1\end{pmatrix}\begin{pmatrix}a_t\\ a_{t+1}\end{pmatrix}=\begin{pmatrix}b+a_{t-1}-g_{t-1}\\ b+\rho a_{t+2}-g_t\end{pmatrix}. \tag{1c}$$

[9] Alternatively, one can assume that the government announces the counterterrorism effort rates, $\{\varphi_t\}_{t=0,\ldots,\infty}$. Since $g_t=\varphi_t a_t$ for $\forall t$, this alternative game yields the same subgame-perfect Nash equilibrium as the one examined in the text. It is more convenient to treat the effort levels rather than the effort rates as the variables controlled by the government.

Let J denote the Jacobian matrix of coefficients of the linear system (1c). Note that J is positive definite. Its determinant, $|J|$, is equal to $1-\rho>0$. The positive definiteness of J implies that the solution to the linear system (1c) is unique and the sufficient condition for the maximization problem is satisfied. By Cramer's rule, the solution of the linear system (1c) is given by

$$a_t = \left[b\left(1+\rho\right)+a_{t-1}+\rho^2 a_{t+2}-g_{t-1}-\rho g_t\right]\Big/\left(1-\rho\right), \tag{1d}$$

$$a_{t+1} = \left[2b+a_{t-1}+\rho a_{t+2}-g_{t-1}-g_t\right]\Big/\left(1-\rho\right). \tag{1e}$$

Equations (1d) and (1e) reveal that the optimal choices for any two adjacent periods are increasing in past and future attack levels and decreasing in current and past counterterrorism effort levels. The intertemporal effects are easy to explain. Past and future attack levels yield positive effects because of habit formation. Attacks in the current and next periods will thus bring future benefits (beyond next period) and the anticipation of such benefits motivate the terrorist organization to supply a higher level of attacks in the current and next periods. As for the intertemporal effects caused by counterterrorism effort levels, we see that past and current effort levels reduce the levels of current and future attacks. Thus, they lower the net benefits associated with the legacy of terror. The linearity of the transformed terrorist organization's payoff function with respect to the benefits and costs of supplying past and contemporaneous attack levels implies that the terrorist organization optimal attack levels are not functions of future counterterrorism effort levels.[10]

[10] Had we considered the first order conditions with respect to the levels of successful attacks rather than with respect to the attack quantities, the successful-attack response functions would have been $s_t = \left[\left(1+\rho\right)b+s_{t-1}+\rho^2\left(s_{t+2}+g_{t+2}\right)-g_t\right]\Big/\left(1-\rho\right)$ and $s_{t+1} = \left[2b+s_{t-1}+\rho\left(s_{t+2}+g_{t+2}\right)-\left(1-\rho\right)g_{t+1}-g_t\right]\Big/\left(1-\rho\right)$. They reveal that successful attacks are increasing functions of g_{t+2}, but decreasing functions of g_t and g_{t+1}. Since $s_t = a_t - g_t$, $s_{t+1} = a_{t+1} - g_{t+1}$ and $s_{t+2} = a_{t+2} - g_{t+2}$, the success-attack response functions are isomorphic to the attack response functions (1d) and (1e).

Multiplying equation (1d) by -1 and adding to equation (1e) yields

$$a_{t+1} - a_t = b + \rho a_{t+2} - g_t . \tag{1f}$$

Equation (1f) informs us how (optimal) attacks evolve over time.[11] By rewriting equation (1f) as $a_{t+2} = \left(a_{t+1} - a_t + g_t - b\right)/\rho$, one important implication of habit formation becomes very clear: the level of attacks in period $t+2$, a_{t+2}, increases with evolution of attacks from period t to period $t+1$, $a_{t+1} - a_t$, as well as with the level of counterterrorism effort supplied by the government at period t, g_t. As we will see below, this feature of our model implies that the level of attacks is an increasing function of counterterrorism effort in the steady state. The level of attacks rises with counterterrorism effort in the steady state at a rate $1/\rho > 1$.

The government's problem is the choice of nonnegative $\left\{g_t, g_{t+1}, a_{t+1}\right\}_{t=0,\ldots,\infty}$ to minimize $\sum_{t=0}^{\infty} \rho^t \left[g_t^2 + \left(a_t - g_t\right)^2 \right]$ subject to the attack evolution equation (1f). The Lagrangian is

$$\sum_{t=0}^{\infty} \rho^t \left[g_t^2 + \left(a_t - g_t\right)^2 \right] - \lambda_t \left(a_{t+1} - a_t - b - \rho a_{t+2} + g_t\right) - \lambda_{t+1}\left(a_{t+2} - a_{t+1} - b - \rho a_{t+3} + g_{t+1}\right).$$

The first order conditions are

$$2\rho^t \left(2g_t - a_t\right) - \lambda_t = 0 , \tag{1g}$$

$$2\rho^{t+1} \left(2g_{t+1} - a_{t+1}\right) - \lambda_{t+1} = 0 , \tag{1h}$$

$$2\rho^{t+1} \left(a_{t+1} - g_{t+1}\right) - \lambda_t + \lambda_{t+1} = 0 . \tag{1i}$$

Combining equations (1g), (1h) and (1i) in order to eliminate the Lagrangian multipliers yields

$$2g_t - a_t = \rho g_{t+1} . \tag{1j}$$

---

[11] See Barros et al (2007) and Faria and Arce (2005, 2011b) for alternative frameworks that rationalize the equation that governs the evolution of terrorist activity in a nation.

The rationale for equation (1j) is straightforward. The left hand side of this equation gives us the net marginal social loss of combating terrorism in period t. The right hand side of the equation provides us with the effective net marginal social loss of combating terrorism in period $t+1$ when this quantity is evaluated from the perspective of period t - it is discounted by ρ.

In the steady state, $g_t = g$ and $a_t = a$ for all t. In the steady state, the equilibrium equations (1f) and (1j) yield $a = (g-b)/\rho$ and $g = a/(2-\rho)$, respectively, provided that $g > b$ (which we have implicitly assumed, since this is required for an interior solution to the problem solved by the government). Hence, in the steady state, the terror organization supplies attacks according to the attack-supply function $\hat{a}(g) = \max\{(g-b)/\rho, 0\}$, for $g \in \mathbb{R}_+$, and the government supplies counterterrorism effort according to the effort-supply function $\hat{g}(a) = a/(2-\rho)$, for $a \in \mathbb{R}_+$. In the relevant range, where $g > b$, the attack- supply function is positively sloped and its slope is large than one. In the relevant range, each extra unit of counterterrorism effort supplied by the government leads to a response of $1/\rho > 1$ units of terror activity by the terrorist organization. In addition, for all $a > 0$, the slope of the government's effort-supply function is positive, but it is less than one: each extra unit of terror activity supplied by the terror organization leads to a response of $1/(2-\rho) < 1$ units of counterterrorism effort by the government. In equilibrium, we obtain:

$$g^* = b/(1-\rho)^2 > b > 0 \text{ , } a^* = (2-\rho)b/(1-\rho)^2 > 0 \text{ and } v^* = s^* = a^* - g^* = b/(1-\rho) > 0 \text{ .} \quad (1k)$$

Results (1k) confirm our earlier claim that the first order conditions for the optimization problems yield interior solutions and a positive level of successful attacks (and thus violence) in the steady state. It is also important to note that in the steady state

$$\varphi^* = g^*/a^* = 1/(2-\rho) > \sigma^* = s^*/a^* = (1-\rho)/(2-\rho) \text{ .} \quad (1l)$$

Thus, the counterterrorism effort rate is higher than the attack success rate, implying that over 50% of attacks supplied by the terrorist organization are avoided by counterterrorism measures.

4. Transnational Counterterrorism Policy Games

Having considered the optimal intertemporal and steady-state counterterrorism policies from the viewpoint of a single nation, we now expand our model to examine policy making from the perspective of two nations whose policy choices are interdependent. The nations are indexed by j, $j = 1, 2$.

The terrorist organization represents a common enemy to both nations. Let $a_{j,t} \geq 0$ denote the number of attacks faced by nation j in period t. The level of successful attacks produced by the terrorist organization in nation j in period t is $s_{j,t} = \sigma_{j,t} a_{j,t}$, where $\sigma_{j,t}$ is the attack-success rate. Let $g_{j,t} = \varphi_{j,t} a_{j,t}$ be the level of counterterrorism effort exerted by the government of nation j in period t, where $\varphi_{j,t}$ is the counterterrorism effort rate.

A nation's counterterrorism effort may benefit or harm the other nation. We restrict our attention to positive external effects. We assume that $s_{j,t} = a_{j,t} - g_{j,t} - \theta g_{k,t}$, where the parameter θ captures the positive marginal external benefit that the counterterrorism effort of nation k produces in nation j, $j, k = 1, 2,\ j \neq k$. To guarantee interior solutions in all games examined below, we assume that $\theta \in [0, 1/2]$. The assumption that $\theta \leq 1/2$ implies that a nation's counterterrorism effort produces a marginal reduction in successful attacks in its soil at least twice as large as in the other nation's soil. Note that if $a_{1,k} > 0$ and $a_{2,t} > 0$, we have $1 = \sigma_{j,t} + \varphi_{j,t} + \theta \varphi_{k,t} a_{k,t} / a_{j,t}$.

In period t, we assume that the utility of the terrorist organization is

$$u\left(s_{1,t}, s_{2,t}, s_{1,t-1}, s_{2,t-1}, y_t; g_{1,t}, g_{2,t}\right) = y_t + b\left(s_{1,t} + s_{2,t}\right) + s_{1,t-1}\left(s_{1,t} + g_{1,t} + \theta g_{2,t}\right) + s_{2,t-1}\left(s_{2,t} + g_{2,t} + \theta g_{1,t}\right).$$

The marginal utility associated with a successful attack in nation j in period t is $b + s_{j,t-1}$ while the marginal utility associated with a successful attack in nation j in period $t-1$, evaluated in terms of the utility at period t, is $s_{j,t} + g_{j,t} + \theta g_{k,t}$. These indicate that the utility function is characterized by adjacent complementarity (habit formation) and that the intertemporal incentives to produce contemporaneous successful attacks in any nation increase with future counterterrorism efforts of both nations. The amount of numeraire good consumed by the international organization is $y_t = \bar{y} - \left(a_{1,t}^2 + a_{2,t}^2\right)/2$. As before, we assume that $\bar{y}$ is sufficiently large so that $y_t \geq 0$, $\forall t$, in all games examined below.

Having observed both nations' policy announcements, the terrorist organization chooses nonnegative $\left\{a_{1,t}, a_{2,t}\right\}_{t=0,\ldots,\infty}$ to maximize[12]

$$\sum_{t=0}^{\infty} \rho^t \left\{ a_{1,t}\left(b + a_{1,t-1} - g_{1,t-1} - \theta g_{2,t-1}\right) + a_{2,t}\left(b + a_{2,t-1} - g_{2,t-1} - \theta g_{1,t-1}\right) - \frac{a_{1,t}^2 + a_{2,t}^2}{2} \right\}.$$

The first order conditions yield, for $j,k = 1,2$, $j \neq k$

$$a_{j,t} - \rho a_{j,t+1} = b + a_{j,t-1} - g_{j,t-1} - \theta g_{k,t-1}, \tag{2a}$$

$$-a_{j,t} + a_{j,t+1} = b + \rho a_{j,t+2} - g_{j,t} - \theta g_{k,t}. \tag{2b}$$

The linear system of equations (2a) and (2b) can be written in matrix form as follows:

[12] The payoff function for the terrorist organization is

$$\bar{m} + \sum_{t=0}^{\infty} \rho^t \left\{ a_{1,t}\left(b + a_{1,t-1} - g_{1,t-1} - \theta g_{2,t-1}\right) + a_{2,t}\left(b + a_{2,t-1} - g_{2,t-1} - \theta g_{1,t-1}\right) - \frac{a_{1,t}^2 + a_{2,t}^2}{2} - b\left(1+\theta\right)\left(g_{1,t} + g_{2,t}\right) \right\},$$

where $\bar{m} = \bar{y}/\left(1-\rho\right)$ if $a_{j,t} > 0$, $j = 1,2$, $\forall t$. Since the terrorist organization takes $g_{j,t}$ as given for $j = 1,2$ and $\forall t$, the term $b\left(1+\theta\right)\left(g_{1,t} + g_{2,t}\right)$ is constant and can be omitted from the relevant objective function.

$$\begin{pmatrix} 1 & -\rho \\ -1 & 1 \end{pmatrix}\begin{pmatrix} a_{j,t} \\ a_{j,t+1} \end{pmatrix} = \begin{pmatrix} b + a_{j,t-1} - g_{j,t-1} - \theta g_{k,t-1} \\ b + \rho a_{j,t+2} - g_{j,t} - \theta g_{k,t} \end{pmatrix}. \tag{2c}$$

As it is clear in equations (2a) – (2b), the maximization problem for the terrorist organization is completely separable in attacks across the targeted nations. Effectively, the terrorist organization solves two separate maximization problems, one for each targeted nation. For each j, the determinant of the Jacobian matrix of system (2c) equals $1-\rho > 0$, implying that the Jacobian matrix is positive definite. This leads us to conclude that the sufficient second order condition is satisfied and the solution to the system of equations (2c) is unique. The solution is as follows:

$$a_{j,t} = \left[b(1+\rho) + a_{j,t-1} + \rho^2 a_{j,t+2} - g_{j,t-1} - \rho g_{j,t} - \theta\left(g_{k,t-1} + \rho g_{k,t} \right) \right] \Big/ (1-\rho), \tag{2d}$$

$$a_{j,t+1} = \left[2b + a_{j,t-1} + \rho a_{j,t+2} - g_{j,t-1} - g_{j,t} - \theta\left(g_{k,t-1} + g_{k,t} \right) \right] \Big/ (1-\rho). \tag{2e}$$

Multiplying equation (2d) by -1 and adding to equation (2e) yields

$$a_{j,t+1} - a_{j,t} = b + \rho a_{j,t+2} - g_{j,t} - \theta g_{k,t}, \qquad j,k = 1,2, \qquad j \neq k. \tag{2f}$$

Equations (2f) provide us with the optimal rules that govern the evolutions of attacks in both nations. They incorporate the terrorist organization's intertemporal best responses to counterterrorism efforts promoted by the two nations. The intertemporal difference in attacks in nation j, $a_{j,t+1} - a_{j,t}$, is more sensitive to this nation's counterterrorism effort than to nation k's: it decreases at a one-to-one rate with respect to $g_{j,t}$ and at a θ-rate with respect to $g_{k,t}$. Due to the transnational external effects associated with counterterrorism effort levels, the evolutions are interdependent, implying that the transnational effects are both contemporaneous and intertemporal.

4.1. Coordinated Transnational Counterterrorism Policy Making

As a benchmark for future comparisons, we first examine the constrained globally efficient allocation which results from minimizing the sum of the nations' intertemporal loss functions subject to the evolution of attacks in each nation. This problem is mathematically identical to the problem solved by an international coalition consisting of both nations which makes fully coordinated choices of counterterrorism efforts accounting for the optimal responses of the terrorist organization.

The Lagrangian for the international coalition is

$$\begin{aligned}
&\sum_{t=0}^{\infty} \rho^t \left[g_{1,t}^2 + g_{2,t}^2 + \left(a_{1,t} - g_{1,t} - \theta g_{2,t}\right)^2 + \left(a_{2,t} - g_{2,t} - \theta g_{1,t}\right)^2 \right] \\
&\quad - \lambda_{1,t}\left(a_{1,t+1} - a_{1,t} - b - \rho a_{1,t+2} + g_{1,t} + \theta g_{2,t}\right) - \lambda_{1,t+1}\left(a_{1,t+2} - a_{1,t+1} - b - \rho a_{1,t+3} + g_{1,t+1} + \theta g_{2,t+1}\right) \\
&\quad - \lambda_{2,t}\left(a_{2,t+1} - a_{2,t} - b - \rho a_{2,t+2} + g_{2,t} + \theta g_{1,t}\right) - \lambda_{2,t+1}\left(a_{2,t+2} - a_{2,t+1} - b - \rho a_{2,t+3} + g_{2,t+1} + \theta g_{1,t+1}\right).
\end{aligned}$$

Since the objective function is strictly convex in all arguments and the constraints are linear, the first order conditions are necessary and sufficient for a global minimum. As we shall demonstrate below, the solution, when evaluated in the steady state, is strictly positive and finite. The first order conditions yield the following equations:

$$2\rho^t \left[g_{1,t} - \left(a_{1,t} - g_{1,t} - \theta g_{2,t}\right) - \theta\left(a_{2,t} - g_{2,t} - \theta g_{1,t}\right)\right] = \lambda_{1,t} + \theta\lambda_{2,t}\,, \tag{3a}$$

$$2\rho^t \left[g_{2,t} - \left(a_{2,t} - g_{2,t} - \theta g_{1,t}\right) - \theta\left(a_{1,t} - g_{1,t} - \theta g_{2,t}\right)\right] = \theta\lambda_{1,t} + \lambda_{2,t}\,, \tag{3b}$$

$$2\rho^{t+1} \left[g_{1,t+1} - \left(a_{1,t+1} - g_{1,t+1} - \theta g_{2,t+1}\right) - \theta\left(a_{2,t+1} - g_{2,t+1} - \theta g_{1,t+1}\right)\right] = \lambda_{1,t+1} + \theta\lambda_{2,t+1}\,, \tag{3c}$$

$$2\rho^{t+1} \left[g_{1,t+1} - \left(a_{1,t+1} - g_{1,t+1} - \theta g_{2,t+1}\right) - \theta\left(a_{2,t+1} - g_{2,t+1} - \theta g_{1,t+1}\right)\right] = \theta\lambda_{1,t+1} + \lambda_{2,t+1}\,, \tag{3d}$$

$$2\rho^{t+1}\left(a_{1,t+1} - g_{1,t+1} - \theta g_{2,t+1}\right) = \lambda_{1,t} - \lambda_{1,t+1}\,, \tag{3e}$$

$$2\rho^{t+1}\left(a_{2,t+1} - g_{2,t+1} - \theta g_{1,t+1}\right) = \lambda_{2,t} - \lambda_{2,t+1}\,. \tag{3f}$$

Equation (3a) informs us that the level of counterterrorism effort supplied by nation 1 that minimizes the constrained collective loss function for the international coalition at period t is the level at which the coalition's net marginal loss from providing counterterrorism effort equates the weighed sum of shadow costs for the constraints that govern the evolutions of attacks in both nations at period t. Equation (3b) has a similar interpretation for the optimal level of counterterrorism effort supplied by nation 2 at period t and equations (3c) and (3d) are the counterparts of equations (3a) and (3b), respectively, for period $t+1$. Equation (3e) states that the optimal level of attacks in nation 1 at period $t+1$ is the level at which the marginal harm from violence faced by this nation equates the intertemporal difference in the shadow costs of the constraints. The international coalition wishes to minimize attacks in nation 1, but it must account for the optimal intertemporal responses of the terrorist organization. A reduction in the number of attacks at $t+1$ relaxes the constraint at this period but tightens the constraint at t. This is the rationale for the intertemporal difference in the shadow costs. Since the marginal harm from violence faced by nation 1 corresponds to this nation's marginal benefit of reducing violence, equation (3e) informs us that the optimal rule equates the marginal benefit of reducing violence at period $t+1$ to the net intertemporal cost of doing so. The interpretation for equation (3f) is similar to the one for equation (3e) and thus it deserves no further comment.

Combining equations (3a) and (3b) yields

$$\lambda_{j,t} = 2\rho^{t}\left[\left(2-\theta^{2}\right)g_{j,t} - \theta^{3}g_{k,t} - \left(1-\theta^{2}\right)a_{j,t}\right] \Big/ \left(1-\theta^{2}\right), \; j,k=1,2, \; j \neq k. \tag{3g}$$

Similarly, combining equations (3c) and (3d) yields

$$\lambda_{j,t+1} = 2\rho^{t+1}\left[\left(2-\theta^{2}\right)g_{j,t+1} - \theta^{3}g_{k,t+1} - \left(1-\theta^{2}\right)a_{j,t+1}\right] \Big/ \left(1-\theta^{2}\right), \; j,k=1,2, \; j \neq k. \tag{3h}$$

Combining equations (3e) - (3h), we obtain for $j,k=1,2,\ \ j\neq k$:

$$\begin{aligned}2\rho^{t+1}\left(1-\theta^2\right)\left(a_{j,t+1}-g_{j,t+1}-\theta g_{k,t+1}\right)=2\rho^{t}\left[\left(2-\theta^2\right)g_{j,t}-\theta^3 g_{k,t}-\left(1-\theta^2\right)a_{j,t}\right]\\-2\rho^{t+1}\left[\left(2-\theta^2\right)g_{j,t+1}-\theta^3 g_{k,t+1}-\left(1-\theta^2\right)a_{j,t+1}\right].\end{aligned} \tag{3i}$$

Equation (3i) informs us that the optimal intertemporal policy rules adopted by the international coalition equate each nation's marginal benefit of reducing violence at period $t+1$ to the nation's net intertemporal marginal cost of providing counterterrorism effort.

In the steady state, where $a_{j,t}=a_j$ and $g_{j,t}=g_j$ for all t, the coordinated equilibrium is determined by the following equations, for $j,k=1,2$, $j\neq k$:

$$g_j+\theta g_k-\rho a_j=b\,, \tag{3j}$$

$$\left(\rho+\theta^2-2\right)g_j+\theta\left(\theta^2-\rho\right)g_k+\left(1-\theta^2\right)a_j=0\,. \tag{3k}$$

Equations (3j) and (3k) follow from equations (2f) and (3i), respectively. Due to the symmetry of our model, the solution to equations (3j) and (3k) is symmetric; namely, $a_j=a$ and $g_j=g$ for $j=1,2$. For future reference, note that the coordinated equilibrium equations simplify to

$$g=\left(b+\rho a\right)\big/\left(1+\theta\right), \tag{3l}$$

$$g=\left(1+\theta\right)a\Big/\left[\left(1+\theta\right)^2+1-\rho\right]. \tag{3m}$$

Equations (3l) and (3m) follow from equations (3j) and (3k), respectively. As in the previous section, the reader is invited to view equations (3l) and (3m) as the expressions that yield the steady-state supply functions of attack and counterterrorism effort levels, respectively. Letting a^{**} and g^{**} denote the quantities that solve equations (3l) and (3m), we obtain

$$a_j^{**}=a^{**}=b\left[\left(1+\theta\right)^2+1-\rho\right]\Big/\left(1-\rho\right)\left[\left(1+\theta\right)^2-\rho\right]>0\,, \tag{3n}$$

$$g_j^{**} = g^{**} = b(1+\theta)\Big/(1-\rho)\left[(1+\theta)^2 - \rho\right] > 0\,, \tag{3o}$$

$$s_j^{**} = s^{**} = v^{**} = a^{**} - (1+\theta)\,g^{**} = b\Big/\left[(1+\theta)^2 - \rho\right] > 0\,. \tag{3p}$$

Note that if $\theta = 0$ we have $a^{**} = a^{*}$, $g^{**} = g^{*}$ and thus $v^{**} = v^{*}$. This is natural because when the spillover index is zero the optimization problem faced by the international coalition can be separated into two independent constrained minimization problems, one for each nation.

The relevant comparative statics are

$$da^{**}/d\theta = -2b(1+\theta)\Big/(1-\rho)\left((1+\theta)^2 - \rho\right)^2 < 0\,, \tag{3q}$$

$$dg^{**}/d\theta = -b\left((1+\theta)^2 + \rho\right)\Big/(1-\rho)\left((1+\theta)^2 - \rho\right)^2 < 0\,, \tag{3r}$$

$$dv^{**}/d\theta = -2b(1+\theta)\Big/\left((1+\theta)^2 - \rho\right)^2 < 0\,. \tag{3s}$$

Results (3q) - (3s) reveal that, in the coordinated equilibrium, terrorist activity, government counterterrorism effort and violence decrease with the spillover factor.

4.2. Uncoordinated Counterterrorism Policy Games

We now consider two types of simultaneous uncoordinated counterterrorism policy games, depending on whether the nations are myopic or farsighted. We say that a nation is myopic if it only accounts for the impact caused by its provision of counterterrorism effort on the evolution of attacks that occurs within its national boundaries, taking the other nation's provision of counterterrorism efforts and their impacts as given. A nation is farsighted if it accounts for the impacts caused by its provision of counterterrorism effort on the evolution of terrorist activity in each nation, including the feedback effects produced on the provision of counterterrorism efforts provided by the other nation. Thus, a farsighted nation does not take the other nation's counterterrorism efforts as given.

4.2.1. Myopic Governments

Each myopic government minimizes its loss function subject to the evolution of terrorist activity in its nation, taking both the choices of the other myopic government and the evolution of terrorist activity that occurs in the other nation as given. The Lagrangian for nation j is

$$L_j^m = \sum_{t=0}^{\infty} \rho^t \left[g_{j,t}^2 + \left(a_{j,t} - g_{j,t} - \theta g_{k,t} \right)^2 \right] - \mu_{j,t} \left(a_{j,t+1} - a_{j,t} - b - \rho a_{j,t+2} + g_{j,t} + \theta g_{k,t} \right)$$
$$- \mu_{j,t+1} \left(a_{j,t+2} - a_{j,t+1} - b - \rho a_{j,t+3} + g_{j,t+1} + \theta g_{k,t+1} \right),$$

$j,k = 1,2$, $j \neq k$. The first order conditions are

$$2\rho^t \left[g_{j,t} - \left(a_{j,t} - g_{j,t} - \theta g_{k,t} \right) \right] = \mu_{j,t}, \tag{4a}$$

$$2\rho^{t+1} \left[g_{j,t+1} - \left(a_{j,t+1} - g_{j,t+1} - \theta g_{k,t+1} \right) \right] = \mu_{j,t+1}, \tag{4b}$$

$$2\rho^{t+1} \left[a_{j,t+1} - g_{j,t+1} - \theta g_{k,t+1} \right] = \mu_{j,t} - \mu_{j,t+1}. \tag{4c}$$

Equation (4a) informs us that the optimal level of counterterrorism effort at period t for nation j is the level that equates this nation's net marginal loss to the shadow cost of the periodic constraint which describes the evolution of terrorist activity in this nation. Equation (4b) describes the optimal rule for counterterrorism effort at period $t+1$; its interpretation is similar to equation (4a). Equation (4c) states that the optimal level of terrorist activity at period $t+1$ for nation j is the level that equates this nation's marginal harm from violence caused by terrorist activity in such a period to the intertemporal difference in the shadow values of the constraints for periods t and $t+1$. Nation j wishes to minimize the level of terrorist activity at period $t+1$

but it is constrained by the conditions that characterize the terrorist organization's optimal responses. A reduction in terrorist activity at $t+1$ relaxes the constraint describing the evolution of terrorist activity in such a period, but it tightens the constraint describing the evolution of terrorist activity in period t. This is the rationale for the intertemporal difference in the shadow costs of the constraints. Since the marginal harm from violence corresponds to the marginal benefit of reducing violence, equation (4c) informs us that the optimal rule for terrorist activity at period $t+1$ for nation j equates this nation's marginal benefit of reducing violence in such a period to the intertemporal net marginal cost of doing so.

Combining equations (4a) – (4c) yields the following system of dynamic equations:

$$a_{j,t} - g_{j,t} - \theta g_{k,t} = g_{j,t} - \rho g_{j,t+1} . \tag{4d}$$

Equations (4d) inform us that nation j's optimal intertemporal policy equates this nation's marginal benefit of reducing violence at period t to the net intertemporal marginal cost of providing counterterrorism effort.

In the steady state, $g_{j,t+1} = g_{j,t} = g_j$ and $a_{j,t+1} = a_{j,t} = a_j$, $j = 1,2$. Given the symmetry of our model, the steady state myopic equilibrium features $g_j = g$ and $a_j = a$, $j = 1,2$. For future reference, note that the equations that characterize the myopic equilibrium are (3l) and

$$g = a \big/ \left(2 + \theta - \rho\right) . \tag{4e}$$

Equation (4e) follows from equation (4d). Letting a^m and g^m denote the myopic equilibrium quantities, we have

$$a^m = b\left(2 + \theta - \rho\right) \big/ \left(1 - \rho\right)\left(1 + \theta - \rho\right) > 0 , \tag{4f}$$

$$g^m = b \big/ \left(1 - \rho\right)\left(1 + \theta - \rho\right) > 0 , \tag{4g}$$

$$v^m = a^m - \left(1 + \theta\right) g^m = b \big/ \left(1 + \theta - \rho\right) > 0 . \tag{4h}$$

Comparing results (3l) - (3n) with results (4f) - (4h) yields:

Proposition 1. For $\theta > 0$, $a^m > a^{**}$, $g^m > g^{**}$ and $v^m > v^{**}$.

Proof. First, $a^m > a^{**}$ if and only if $(2+\theta-\rho)\left((1+\theta)^2-\rho\right) > \left(1+(1+\theta)^2-\rho\right)(1+\theta-\rho)$. The latter inequality holds if and only if $\theta > 0$. Second, $g^m > g^{**}$ if and only if $(1+\theta)^2-\rho > (1+\theta)(1+\theta-\rho)$. The latter inequality holds if and only if $\rho\theta > 0$. Finally, $v^m > v^{**}$ if and only if $(1+\theta)^2-\rho > (1+\theta-\rho)$. The latter holds if and only if $\theta > 0$. ■

The myopic nations ignore both contemporaneous and future transnational spillover benefits when they make their decisions. The comparison between equations (4a) and (4b) with their respective counterparts in the set of equations (3a) - (3d) makes it clear how the inefficient behavior of the myopic nations deviates from the behavior of the international coalition, which internalizes all spillover benefits. This differential in behavior is also captured by the discrepancy between equations (3m) and (4e). This is the sole distinction between the equations that determine the equilibria, since both equilibria satisfy equation (3l). In the $\mathbb{R}_+^2$ space, consider the variable g as a function of the variable a (thus, the values for g are displayed in the vertical axis and the values for a are in the horizontal axis). The curves that characterize the coordinated equilibrium are

$$g = \varphi(a) = (b+\rho a)/(1+\theta), \tag{4i}$$

$$g = \psi^{**}(a) = (1+\theta)a \Big/ \left[(1+\theta)^2 + 1 - \rho\right]. \tag{4j}$$

The curves that characterize the myopic equilibrium are (4i) and

$$g = \psi^m(a) = a/(2+\theta-\rho). \tag{4k}$$

Since equations (4j) and (4k) are identical to equations (3m) and (4e), respectively, the discrepancy between equations (3m) and (4e) is captured by the discrepancy between equations (4j) and (4k). Note that the slope of $\psi^{**}(a)$, $(1+\theta)\big/\left[(1+\theta)^2+1-\rho\right]$, is greater than the slope of $\psi^m(a), 1/(2+\theta-\rho)$, for all $a \geq 0$ because $\theta > 0$. The slope of $\psi^{**}(a)$ is greater than the slope of $\psi^m(a)$ due to the fact that the spillover effects are fully internalized in the coordinated equilibrium. This fact leads to the conclusion that $a^m > a^{**}$ (this result would hold were $\varphi(a)$ decreasing, constant or increasing in a). Since $\varphi'(a) > 0$ for all $a \geq 0$, we also have $g^m > g^{**}$. Thus, the result that the myopic equilibrium features overprovision of counterterrorism effort follows from the positive slope of the terror-supply function, $\varphi(a)$. As we pointed out earlier, the positive slope of this supply function derives its rationale from the fact that the terrorist organization cares about the legacy of terror (which is an immediate consequence of habit formation). In sum, as described by Proposition 1, the myopic equilibrium is characterized by higher terrorist and counterterrorist activities than the coordinated equilibrium.

4.2.2. Farsighted Governments

Farsighted governments make policy decisions in full anticipation of the evolution of terrorist activities in both nations. Each nation can see that its provision of counterterrorism efforts affects the evolution of terrorist activity in the other nation because of the positive spillovers in counterterrorism and that their efforts promote feedback effects through the other nation's provision of counterterrorism efforts. Each nation internalizes the feedback effects in its optimization problem. In this sense, each nation behaves as the international coalition. Formally, the constraint set faced by each nation in its minimization problem is identical to the constraint set for the problem solved by the international coalition. Were, in addition, each farsighted nation altruistic, caring about the social costs incurred by the other nation as much as they care about their own social costs, the problem that would be solved by each farsighted nation would be mathematically identical to the problem solved by the international coalition. However, each nation only cares about its loss function, as in the myopic setting. Thus, the farsighted nations can be understood as a hybrid of myopic and fully coordinated governments.

Consider the equations that govern the evolutions of terrorist activities:

$$a_{1,t+1} - a_{1,t} = b + \rho a_{1,t+2} - g_{1,t} - \theta g_{2,t}, \tag{5a}$$

$$a_{2,t+1} - a_{2,t} = b + \rho a_{2,t+2} - g_{2,t} - \theta g_{1,t}. \tag{5b}$$

Solving equation (5a) for $g_{1,t}$ we can write

$$g_{1,t}\left(g_{2,t}; a_{1,t}, a_{1,t+1}, a_{1,t+2}, b, \theta\right) = b + \rho a_{1,t+2} - \left(a_{1,t+1} - a_{1,t}\right) - \theta g_{2,t}. \tag{5c}$$

Equation (5c) informs us that the contemporaneous counterterrorism effort of nation 1 is decreasing at one-to-one rate with the evolution of terrorism activity in this nation and also decreasing at a θ-rate with the level of counterterrorism effort supplied by nation 2. Similarly, if we solve equation (5b) for $g_{2,t}$, we can write[13]

$$g_{2,t}\left(g_{1,t}; a_{2,t}, a_{2,t+1}, a_{2,t+2}, b, \theta\right) = b + a_{2,t} + \rho a_{2,t+2} - a_{2,t+1} - \theta g_{1,t}. \tag{5d}$$

Inserting $g_{2,t}\left(g_{1,t}; a_{2,t}, a_{2,t+1}, a_{2,t+2}, b, \theta\right)$ into equation (5a) enables us to write the evolution of terror activity in nation 1 from period t to period $t+1$ as a function of subsequent terror activity, $a_{1,t+2}$, the nation's contemporaneous counterterrorism effort, $g_{1,t}$, the other nation's

[13] It is important to note that the functions described by equations (5c) and (5d) are derived from the terrorist organization's best-response functions. They do not represent the nations' best-response functions. They should be understood as "feedback functions". The nations' best-response functions are determined by taking the feedback functions into account.

evolution of terror activity from period t to period $t+1$, $a_{2,t+1}-a_{2,t}$, and the other nation's subsequent terror activity, $a_{2,t+2}$:

$$a_{1,t+1}-a_{1,t}=(1-\theta)b+\rho a_{1,t+2}-\left(1-\theta^2\right)g_{1,t}+\theta\left(a_{2,t+1}-a_{2,t}\right)-\theta\rho a_{2,t+2}. \tag{5e}$$

If nation 1 anticipates that the evolution of terror activity in nation 2 leads to the relationship described by $g_{2,t}\left(g_{1,t};a_{2,t},a_{2,t+1},a_{2,t+2},b,\theta\right)$ and fully accounts for the impact of this relationship on the evolution of terror activity in its own soil, it correctly anticipates that the evolution of terror activity in its soil is described by equation (5e). Among other things, equation (5e) describes the evolution of terror activity in nation 1 as a function of direct and feedback (through $g_{2,t}\left(g_{1,t};a_{2,t},a_{2,t+1},a_{2,t+2},b,\theta\right)$) effects of the contemporaneous counterterrorism effort exerted by nation 1. This nation now clearly sees that its level of counterterrorism effort in period t, $g_{1,t}$, reduces the evolution of terrorist activity in its soil from period t to period $t+1$, $a_{1,t+1}-a_{1,t}$, at a rate $\left(1-\theta^2\right)<1$. When nation 1 is myopic, it perceives a unitary rate of reduction in the evolution of terrorist activity in its soil promoted by its provision of counterterrorism effort in each period. It is, therefore, clear that a farsighted nation perceives its counterterrorism policy as being less effective in reducing violence in its soil than a myopic nation.

Combining equations (5b) and (5c), we have

$$a_{2,t+1}-a_{2,t}=(1-\theta)b+\rho a_{2,t+2}+\theta\left(a_{1,t+1}-a_{1,t}-\rho a_{1,t+2}\right)-\left(1-\theta^2\right)g_{2,t}. \tag{5f}$$

Equation (5f) represents nation 2's perception of the evolution of terrorist activity in its soil.

The loss perceived by a farsighted nation j at period t is $z_j^f\left(g_{j,t},a_{j,t};a_{k,t},a_{k,t+1},a_{k,t+2}\right)=g_{j,t}^2+\left[a_{j,t}-\left(1-\theta^2\right)g_{j,t}-\theta\left(b+a_{k,t}+\rho a_{k,t+2}-a_{k,t+1}\right)\right]^2$, $\quad j,k=1,2$, $j\neq k$. The Lagrangian for nation j is

$$
\begin{aligned}
L_j^f &= \sum_{t=0}^{\infty} \rho^t \left\{ g_{j,t}^2 + \left[a_{j,t} - \left(1-\theta^2\right) g_{j,t} - \theta\left(b + a_{k,t} + \rho a_{k,t+2} - a_{k,t+1}\right)\right]^2 \right\} \\
&-\eta_{j,t}\left[a_{j,t+1} - a_{j,t} - \rho a_{j,t+2} - (1-\theta) b + \left(1-\theta^2\right) g_{j,t} - \theta\left(a_{k,t+1} - a_{k,t} - \rho a_{k,t+2}\right)\right] \\
&-\eta_{j,t+1}\left[a_{j,t+2} - a_{j,t+1} - \rho a_{j,t+3} - (1-\theta) b + \left(1-\theta^2\right) g_{j,t+1} - \theta\left(a_{k,t+2} - a_{k,t+1} - \rho a_{k,t+3}\right)\right],
\end{aligned}
$$

where $\eta_{j,t}$ and $\eta_{j,t+1}$ are the Lagrangian multipliers. For $j,k=1,2$, $j \neq k$, the first order conditions are

$$2\rho^t \left[g_{j,t} - \left(1-\theta^2\right)\left(a_{j,t} - g_{j,t} - \theta g_{k,t}\right)\right] = \left(1-\theta^2\right)\eta_{j,t}, \tag{5g}$$

$$2\rho^{t+1} \left[g_{j,t+1} - \left(1-\theta^2\right)\left(a_{j,t+1} - g_{j,t+1} - \theta g_{k,t+1}\right)\right] = \left(1-\theta^2\right)\eta_{j,t+1}, \tag{5h}$$

$$2\rho^{t+1}\left(a_{j,t+1} - g_{j,t+1} - \theta g_{k,t+1}\right) + \eta_{j,t+1} = \eta_{j,t}. \tag{5i}$$

Equation (5e) informs us that the optimal level of counterterrorism effort in nation j at period t is the level at which this nation's effective net marginal loss from providing counterterrorism effort is equal to effective shadow cost of the periodic constraint which governs the evolution of terrorist activity in nation j at period t. The interpretation for equation (5f) is similar. The interpretation for equation (5g) is identical to the interpretation for the counterparts in the previous optimization problems.

Combining equations (5g) - (5i) yields

$$\left(1-\theta^2\right)\left(a_{j,t} - g_{j,t} - \theta g_{k,t}\right) = g_{j,t} - \rho g_{j,t+1}. \tag{5j}$$

Equations (5j) inform us that nation j's optimal intertemporal policy equates this nation's perceived marginal benefit of reducing violence at period t to the net intertemporal marginal cost of providing counterterrorism effort.

In the steady state, the equilibrium is characterized by the following equations, for $j,k=1,2$, $j \neq k$:

$$\left(1-\theta^2\right)g_j = (1-\theta)b + \rho\left(a_j - \theta a_k\right), \qquad (5k)$$

$$\left(1-\theta^2\right)a_j = \left(2-\rho-\theta^2\right)g_j + \theta\left(1-\theta^2\right)g_k . \qquad (5l)$$

Equations (5k) follow from equations (5e) and (5f) and equations (5l) follow from equations (5j). The farsighted equilibrium is symmetric (i.e., $a_j = a$ and $g_j = g$, $j = 1,2$). The equilibrium quantities solve equation (3l), which also follows from equations (5k) given the symmetry, and

$$g = \left(1-\theta^2\right)a \Big/ \left[(1-\rho) + (1+\theta)\left(1-\theta^2\right)\right]. \qquad (5m)$$

Equation (5m) follows from equation (5l). Letting the superscript " f " denote the quantities in the farsighted equilibrium, we have

$$a_j^f = a^f = b\left[1 + (1+\theta)\left(1-\theta^2\right) - \rho\right] \Big/ (1-\rho)\left[(1+\theta)\left(1-\theta^2\right) - \rho\right] > 0, \qquad (5n)$$

$$g_j^f = g^f = b\left(1-\theta^2\right) \Big/ (1-\rho)\left[(1+\theta)\left(1-\theta^2\right) - \rho\right] > 0, \qquad (5o)$$

for $j = 1,2$. The common denominator in results (5n) and (5o) is positive because

$$(1+\theta)\left(1-\theta^2\right) \geq 1 > \rho$$

for $\theta \in [0,1/2]$. Given results (5n) and (5o), we have $v_j^f = v^f$, for $j = 1,2$, where

$$v^f = b \Big/ \left[(1+\theta)\left(1-\theta^2\right) - \rho\right] > 0 . \qquad (5p)$$

Comparing results (4f) - (4h) with results (5n) - (5p), we obtain

Proposition 2. For $\theta > 0$, $a^f > a^m$, $g^f > g^m$ and $v^f > v^m$.

Proof. For $a^f > a^m$, $\left[1 + (1+\theta)\left(1-\theta^2\right) - \rho\right](1+\theta-\rho) > (2+\theta-\rho)\left[(1+\theta)\left(1-\theta^2\right) - \rho\right]$, which is true if and only $\theta^2 > 0$. Similarly, $g^f > g^m$ and $v^f > v^m$ if and only if $\theta^2 > 0$.

In the $\mathbb{R}_+^2$ space, considering the variable g as a function of the variable a, the curves that characterize the farsighted equilibrium are (4i) and

$$g = \psi^f(a) = (1-\theta^2)a \Big/ \left[1+(1+\theta)(1-\theta^2)-\rho\right].$$

Since the slope of $\psi^f(a)$ is smaller than the slope of $\psi^m(a)$ for all $a \geq 0$ and since $\varphi'(a) > 0$ for all $a \geq 0$, we have $a^f > a^m$ and $g^f > g^m$. The slope of $\psi^f(a)$ being smaller than the slope of $\psi^m(a)$ is consistent with our earlier observations that the farsighted nations perceive their counterterrorism policies as being less effective in reducing violence than the myopic nations.

Combining Propositions 1 and 2, we can write:

Proposition 3. For $\theta > 0$, $a^f > a^m > a^{**}$, $g^f > g^m > g^{**}$ and $v^f > v^m > v^{**}$.

As Proposition 3 makes it clear, the farsighted equilibrium is the least efficient one. Letting z^{**}, z^m and z^f denote the loss amounts faced by each nation in the coordinated, myopic and farsighted equilibria, we can now affirm:

Proposition 4. For $\theta > 0$, $z^f > z^m > z^{**}$.

Proof. Since $z^f = (g^f)^2 + (v^f)^2$, $z^m = (g^m)^2 + (v^m)^2$ and $z^{**} = (g^{**})^2 + (v^{**})^2$, we have $z^f > z^m > z^{**}$ for $\theta > 0$ because $(g^f)^2 > (g^m)^2 > (g^{**})^2$ and $(v^f)^2 > (v^m)^2 > (v^{**})^2$ for $\theta > 0$. ■

5. Discussion and Concluding Remarks

The simplest model examined in this paper, in which there are no transnational spillovers, we demonstrated that a terrorist organization, which cares about the legacy that terror promotes, finds it optimal to supply terrorist activity at a non-decreasing rate with the level of counterterrorism effort supplied by the government in the steady state. In the expanded model, we studied the impact of transnational intertemporal externalities on coordinated and uncoordinated counterterrorism policy making. The games involved three long-lived players, two national governments and an international terrorist organization. We considered two uncoordinated games, one in which both nations are myopic and another in which both nations are farsighted.[14] A myopic nation makes decisions ignoring transnational spillovers. A farsighted nation makes decision fully accounting for the direct and feedback effects that terror promotes in both nations. A farsighted nation, however, does not care about the social costs that originate with terror activity in the other nation.

The main model yields two important results. First, the uncoordinated equilibria are characterized by overprovision of counterterrorism efforts. Overprovision derives its rationale from the fact that the terrorist organization cares about the legacy that terror promotes. The incentives associated with the legacy of terror lead the terrorist organization to react positively to an increase in counterterrorism effort in the steady state; in fact, the rate at which terrorist activity increases in response to an increase in counterterrorism effort in the steady state is greater than one.

Second, the farsighted equilibrium is the least efficient one, featuring higher counterterrorism efforts, higher terrorist activities and higher violence levels than the myopic equilibrium, with the latter featuring higher counterterrorism efforts, higher terrorist activities and higher violence levels than the coordinated equilibrium. The inefficiency of the farsighted equilibrium relative to the myopic equilibrium is due to the fact that each farsighted nation can see that the effectiveness rate of its counterterrorism efforts in reducing the evolution of terrorist activities in its soil is sensitive to the feedback effects that its counterterrorism efforts promote on the other nation's counterterrorism policy. Nation j knows that nation k reduces its

[14] An interesting avenue for future work is to consider a "supergame" in which the nations decide whether to be myopic or farsighted. To formally carry out this analysis, one would have to consider settings in which the nations are asymmetric, with one nation being myopic and the other being farsighted. One of the interesting issues that may be examined in this context is whether being farsighted is a dominant strategy for each nation.

provision of counterterrorism effort in each period by a rate equal to the spillover index θ and also that each unit of counterterrorism effort supplied by nation k in each period yields a spillover benefit also equal to the spillover index θ to nation j. Hence, the marginal feedback effect felt by nation j in each period is a loss of θ^2 units in the effectiveness of its counterterrorism effort in reducing the evolution of terrorist activity in its soil. Thus, a farsighted nation perceives that it must supply a greater level of counterterrorism effort in each period relative to a myopic nation in order to reach the same amount of reduction in the evolution of terrorist activity. The combination of a higher incentive to supply counterterrorism effort in the farsighted setting and the positive response of the terrorist organization associated with an increase in counterterrorism effort lead to the conclusion that the farsighted equilibrium is less efficient than the myopic equilibrium.

References

Arce, D.G. and T. Sandler (2005) Counterterrorism: A game-theoretic analysis, *Journal of Conflict Resolution 49, 183 - 200.*

Barros, C. P. , I. Proença, J. R. Faria, and L. Gil-Alana (2007) Are US citizens at risk of terrorism in Europe? *Defence and Peace Economics* 18, 495-507.

Becker, G.S. and K.M. Murphy (1988) A theory of rational addiction, *Journal of Political Economy* 96, 675-700.

Behrens, D.A., J.P. Caulkins, G. Feichtinger, and G. Tragler (2007) Incentive Stackelberg strategies for a dynamic game on terrorism, *Annals of the International Society of Dynamic Games* 9, Part V, 459-486.

bin Laden, O., A. al-Zawahiri, A.-Y.R.A. Taha, S.M. Hamzah, and F. Rahman (1998) *Jihad Against the Jews and Crusaders*, www.fas.org/irp/world/para/docs/980223-fatwa.htm.

Bloom, M. (2005) *Dying to Kill: The Allure of Suicide Terror,* Columbia University Press: New York.

Boyer, M. (1978) A habit forming optimal growth model, *International Economic Review* 19, 585-609.

Boyer, M. (1983) Rational demand and expenditure patterns under habit formation, *Journal of Economic Theory* 31, 27-53

Cook, D. (2006) The role of Islam as a motivating factor in Usama bin Ladin's appeal within the Muslim world, in D.G. Kamien (Ed.) *The McGraw-Hill Homeland Security Handbook*, McGraw-Hill, New York.

Das, S. (2008) Some mechanisms of terror cycle, *Journal of Economic Behavior & Organization* 67, 644-656.

Dockner, E. J. and G. Feichtinger (1993) Cyclical Consumption Patterns and Rational Addiction. *American Economic Review* 83, 256-263.

Enders, W. and T. Sandler (2006) *The Political Economy of Terrorism*, Cambridge University Press: Cambridge.

Faria, J.R. (2003) Terror cycles, *Studies in Nonlinear Dynamics and Econometrics* 7, No.1, Article 3.

Faria, J.R. and D.G. Arce (2005) Terror support and recruitment, *Defence and Peace Economics* 16, 263-273.

Faria, J.R. and D.G. Arce (2011a) Counterterrorism and its impact on terror support and recruitment: Accounting for backlash, *Defence and Peace Economics*, forthcoming.

Faria, J.R. and D.G. Arce (2011b) A vintage model of terrorist organizations, *Journal of Conflict Resolution*, forthcoming

Feldstein, M. (2008) Designing institutions to deal with terrorism in the United States, *American Economic Review Papers and Proceedings* 98, 122-126.

Feichtinger G. and A.J. Novak (2008) Terror and Counterterror Operations: Differential Game with Cyclical Nash Solution, *Journal of Optimization Theory and Applications* 139, 541-556.

Fox, M., L.P. Tost, K.A. Wade-Benzoni (2010) The legacy motive: A catalyst for sustainable decision making in organizations, *Business Ethics Quarterly* 20, 153-185

Gartenstein-Ross, D. (2011) *Bin Laden's Legacy: Why We're Still Losing the War on Terror*, John Wiley & Sons, Inc., Hoboken, New Jersey.

Hartwick, J.M. (1980) The intertemporal externality in the dynamic common property renewable resource problem, *Economics Letters* 5, 275-280.

Hashim, A.S. (2006) Al-Qaida: Origins, goals, and grand strategy, in D.G. Kamien (Ed.) *The McGraw-Hill Homeland Security Handbook*, McGraw-Hill, New York.

Hoffman, B. (2006) *Inside Terrorism*, Columbia University Press: New York.

Horgan, J. (2005) *The Psychology of Terrorism*, Routledge: London and New York.

Jacobson, D. and E. Kaplan (2007) Suicide Bombings and Targeted Killings in (Counter-) Terror Games, *Journal of Conflict Resolution* 51: 772-792.

Iannaccone, L.R. (1986) Addiction and satiation, *Economics Letters* 21, 95-99.

Lapan H.E., and T. Sandler (1993) Terrorism and signalling, *European Journal of Political Economy* 9, 383-397.

Lee, D. R. (1988) Free riding and paid riding in the fight against terrorism, *American Economic Review Papers and Proceedings* 78, 22-26.

Lee, D. R., and T. Sandler (1989) On the optimal retaliation against terrorists: The paid-rider option, *Public Choice* 62, 141-152.

Novak A.J., G. Feichtinger, and G. Leitmann, (2010) A differential game related to terrorism: Nash and Stackelberg Strategies, *Journal of Optimization Theory and Applications* 144, 533-555.

Rosendorff, P. and T. Sandler (2004) Too much of a good thing? The proactive response dilemma, *Journal of Conflict Resolution* 48, 657-671.

Rozen, K. (2010) Foundations of intrinsic habit formation, *Econometrica* 78, 1341-1373.

Ryder, H. E. and Heal, G. M. (1973). Optimal growth with intertemporally dependent preferences. *Review of Economic Studies* 40, 1-31.

Sandler, T. (2005) Collective versus unilateral responses to terrorism, *Public Choice* 124, 75-93.

Sandler, T., and D. Arce (2003) Terrorism and game theory, *Simulation & Gaming* 34, 319-337.

Sandler, T., and D. Arce (2007) Terrorism: A game-theoretic approach, *Handbook of Defense Economics*, Vol. 2, 775-813.

Sandler, T. and H.E. Lapan (1988) The calculus of dissent: An analysis of terrorists' choice of targets, *Synthese* 76, 245-261.

Sandler, T. and K. Siqueira (2006) Global terrorism: Deterrence versus preemption, *Canadian Journal of Economics* 39, 1370-1387.

Siqueira, K. and T. Sandler (2007) Terrorist backlash, terrorism mitigation, and policy delegation, *Journal of Public Economics* 91, 1800-1815.